ON THE USE OF MARKOV ANALYSIS IN MARKETING OF TELECOMMUNICATION PRODUCT IN NIGERIA

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Abstract:

This paper examined the application of Markov Chain in marketing three competitive networks that provides the same services. Markov analysis has been used in the last few years mainly as marketing, examining and predicting the behaviour of customers in terms of their brand loyalty and their switching from one brand to another. The three networks are Airtel, MTN and Globacom are used as a case study. With the application problem, we examine and answer the question on the proportion of the subscribers that each network have at the end of each month when we assumed the same pattern of gains and losses. We observed that, Airtel has the largest proportion of retaining their subscriber followed by MTN and Globacom in that order. Finally, mean recurrence for each network were also determined.

Keywords: *Markov-Chain, Transition probability, Markov- property* **,** *Equilibrium, Networks and Subscribers.*

Introduction

A Markov- chain is defined as a Markov - process with a discrete state space and a discrete set of time parameter. We also defined the Markov - property as that which possessed by a process whose future probability behaviour is uniquely determined solely by the present state of the system. That is, (X_n) is Markov iff

$$\mathrm{P}\left\{X_{n+1} = x_{x+1} \middle| X_{0=x_0;} X_{1=x_{1,}\ldots\ldots,} X_{n=x_n}\right\} \quad \ldots \ldots \ldots \ldots (1)$$

which gives

$$\mathrm{P}\{X_{n+1=} x_{n+1} | X_{n=} x_n\}.$$

As a management tool, Markov analysis has been used in last few years mainly as marketing and for examining and predicting the behaviour of customers in terms of their brand loyalty and their switching from one brand to another. Application of Markov analysis would not be fully completed in management without an extensive mathematical background.

Other areas in which the application of Markov analysis has produced significant contributions are in accounting by which Markov analysis can be applied to the behaviour of accounts receivable such as , credit customers of a company could be divided arbitrarily into three groups namely , customers who pay promptly, customers who pay only after considerable collection action and customers whose balances are written off as losses.

Based on the past behaviour of the customers in each of these three groups, one can easily establish the transition probabilities from each group to each other group. Efforts have been made by various researchers in the past to study the behaviour of stock market prices using Markov chain model, while a few of them hold the beliefs that contain price trends and pattern exist to enable the investor to make better predictions of the expected values of future stock market price changes, the majority conclude that past price data alone cannot form the basis for predicting the expected values of price movement in the stock market . Some researchers have used this model in different context incorporating new methods for applying model (Kalbfleisch and Lawless, 1985; Kay, 1986; Lu and Stitt, 1994; Gentleman et al, 1994; Satten and Longini, 1996). Non – homogeneous Markov models in the analysis of survival after breast cancer was carried out by Rafael Perez- Ocon et al (2001). The results of the applications of Markovian properties in the temperate forests showed that short- term changes in species composition could be predicted with good accuracy.

Osho (1990) described the application of a continuous Time Markov chain to secondary succession in a Nigeria Tropical moist forest as a pure birth and death process. The result obtained under the continuous time indicated that the under- storey and middle species were leaving the time population at an exponential rate while the top canopy species increased exponentially.

Uche (1982) discusses the development of stochastic modelling of the educational process. It therefore concerns itself with quantitative educational planning using the stochastic model in modelling many aspects or level of education. In usin the Markov chain models , one need to be aware of the limitations of the assumptions as well as the various sources of error. These sources have been classified and later grouped into three major classes. The three classification of errors are the random error, estimation errors and specification errors.

Finally, this paper we examine and gives the proportion of subscribers of each network at the end of each month of operation and on the long-run(i.e .in equilibrium) among others

2.0 Methods of Analysis And Concepts

2.1 Transition probability. The probability of jump from one state x to another state y is called a transition probability from state x to state y denoted by P(x,y).

2.2 Transition probability matrix. Let P_{ij} be the transition probabilities of the given Markov chain. The elements are written in matrix form as

$$P=\begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This matrix is called a transition matrix of the Markov chain. Equation (2) satisfies the following properties.

(i) all the elements of the transition matrix P are probabilities and hence

$P_{ij} \geq 0$ and $0 \leq P_{ij} \leq 1$, i = 1,2 3 ,......., n and j = 1,2,3,.....,m

(ii) The sum of all probabilities in any given row is unity.i.e $\sum_j^m P_{ij} = 1$.

2.3 Stationary Markov Process. A Markov chain is said o be stationary if the transition probabilities are independent of time t. It depends only on the current state E_s and the previous state E_r i.e

$$P(X_{t+1} = E_s | E_{t=} E_r) = \text{Prs}.$$

In this study, we are concerned with the patronage decisions of subscribers; it involves how many subscribers are subscribing from each networks. A basic assumption is that subscribers do not shift their patronage from network to network at random, instead , we assume that the choices of networks to subscribe from in the future reflect choices made in the past.

A first-order Markov process is based on the assumption that the probability of the next event (subscribers' choices of networks next month ,in this case) depends upon the outcomes of the last event (subscribers' choice this month) and not at all on any earlier choices.

A second-order Markov process assumes that subscriber choices next month may depend upon their choices during the immediate past 2 months.

In turn, a third-order process is based upon the assumption that subscribers behaviours is best predicted by observing and taking account of their behaviour during the past 3 months. Some interest in the theory of Markov chain includes the determination of the probability distribution for each random variable Xn and also in the limiting distribution of Xn as n→ ∞ (asymptotic property) given the initial probabilities and the transition probabilities.

Practical illustration of markov analysis in marketing strategy with numerical examples.

On January 1, of the subscribers in Ibadan the Airtel has $\frac{1}{2,}$, the MTN has $\frac{1}{4,}$, and Globacom has $\frac{1}{4}$. During the month of January, the Airtel retains $\frac{7}{8}$ of its subscribers and losses $\frac{1}{8}$ of them to MTN. The MTN retains $\frac{1}{12}$ of its subscribers and losses $\frac{3}{4}$ of them to Airtel. The Globacom retains $\frac{1}{3}$ of its subscribers and losses $\frac{1}{2}$ of them to Airtel and $\frac{1}{6}$ of them to MTN. Assuming there are no new subscribers and that none of the subscribers quit subscribing. We want to determine (i) the proportion of the subscribers that each network have on February 1. (ii). the proportion of subscribers that each network have on March 1, assuming the same pattern of gains and losses continuous for February and (iii) the proportion of subscribers will each network have in the long-run (i. e, in equilibrium)?.

Here, we let 1 represent AIRTEL , 2 represent MTN and 3 represent GLOBACOM. So that , the probability transition matrix P is given by

$$P = \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix} \text{...(3)}$$

For (i) we obtain $P^1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix} = \begin{pmatrix} \frac{3}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$

Interpretations of the above matrix: Row 1 x Column 1 :

Airtel's propensity to retain its subscribers x Airtel's share of subscriber is

$= \frac{7}{8} \text{ x } \frac{1}{2} = \frac{7}{16}$

Airtel's propensity to attract MTN's subscribers x MTN's share of subscriber is

$= \frac{3}{4} \text{ x } \frac{1}{4} = \frac{3}{16}$ And

Airtel's propensity to attract Globacom's share of subscriber is

$= \frac{1}{2} \text{ x } \frac{1}{4} = \frac{1}{8}$.

Hence , the probable February 1 , Airtel shares of subscribers is

$= \frac{7}{16} + \frac{3}{16} + \frac{1}{8} = \frac{3}{4}$ Row 1 Column 2 :

MTN's propensity to attract Airtel's subscribers x Airtel's share of subscriber is

$= \frac{1}{8} \text{ x } \frac{1}{2} = \frac{1}{16}$

MTN's propensity to retain its subscriber x MTN's share of subscriber is

$= \frac{1}{12} \text{ x } \frac{1}{4} = \frac{1}{48}$ And

MTN's propensity to attract Globacom's subscribers x Globacom's share of subscriber is

$= \frac{1}{6} \text{ x } \frac{1}{4} = \frac{1}{24}$

Also, the probable February 1 , MTN shares of subscribers is

$= \frac{1}{16} + \frac{1}{48} + \frac{1}{24} = \frac{1}{8}$ Row 1 x Column 3 :

Globacom's propensity to attract Airtel's subscribers x Airtel's share of subscriber is

$= 0 \text{ x } \frac{1}{2} = 0$

Globacom's propensity to attract MTN's subscribers x MTN's share of subscriber is

$= \frac{1}{6} \text{ x } \frac{1}{4} = \frac{1}{24}$ And

Globacom's propensity to retain its subscribers x Globacom's share of subscribers is

$= \frac{1}{3} \text{ x } \frac{1}{4} = \frac{1}{12}$

Therefore , the probable February 1, Globacom shares of subscribers is

$$= 0 + \frac{1}{24} + \frac{1}{12} = \frac{1}{8}$$

Thus, on February 1 , of the subscribing the Airtel has $\frac{3}{4}$, the MTN has $\frac{1}{8}$ and the Globacom has $\frac{1}{8}$ of subscribers.

(ii) The probable subscriber share on March 1 can be computed by squaring the matrix of transition probabilities and multiplying the squared matrix by January 1 subscriber shares.

That is ,

$$P= \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}^2 = \begin{pmatrix} \frac{13}{16} & \frac{1}{8} & \frac{1}{16} \end{pmatrix} \text{ OR}$$

Multiply the matrix of transition probabilities by the subscribers shares on February 1. That

$$P^2 = \begin{pmatrix} \frac{3}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix} \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix} = \begin{pmatrix} \frac{13}{16} & \frac{1}{8} & \frac{1}{16} \end{pmatrix}$$

Thus, on March 1 of the subscribing the Airtel has $\left(\frac{13}{16}\right)$, the MTN has $\left(\frac{1}{8}\right)$ and Globacom has $\left(\frac{1}{16}\right)$ of subscribers.

In general, this approach can be used to obtain the subscribers shares for day 1 of any months.

That is, P^n , $= \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}^n$, where n = 1, 2, 3,................................6)

(iii) Equilibrium conditions: It is quite reasonable to assume that a state of equilibrium might be reached in the future regarding subscribers shares. That is, the exchange of subscribers under equilibrium would be such as to continue-to-free- the three network shares which obtained at the moment of equilibrium was reached. This equilibrium can result only if no network takes action that alters the matrix of transition probabilities. Hence, the probability of the long- run subscribing will be

uP = u In matrix notation ,i . e.

$$\begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{bmatrix} \frac{7}{8} & \frac{1}{8} & 0 \\ \frac{3}{4} & \frac{1}{12} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix} = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \text{ ....(6)}$$

By expanding equation (6) above, we then have a system of linear equations

$\frac{7}{8}u_1 + \frac{3}{4}u_2 + \frac{1}{2}u_3 = u_1$

$\frac{1}{8}u_1 + \frac{1}{12}u_2 + \frac{1}{6}u_3 = u_2$

$\frac{1}{6}u_2 + \frac{1}{3}u_3 = u_3$

And that , $u_1 + u_2 + u_3 = 1$

By solving the system of linear equations above, we then have ,

$u_1 = \frac{28}{33}$, $u_2 = \frac{4}{33}$ and $u_3 = \frac{1}{33}$.

Conclusion:

It can be concluded that, in the long-run of the subscribers the Airtel will have $\frac{28}{33}$, the MTN will have $\frac{4}{33}$ and the Globacom will have $\frac{1}{33}$. Hence, the mean of recurrence for each network is Airtel = $\frac{33}{28}$ = 1.179 months

MTN = $\frac{33}{4}$ = 8. 25 months

Globacom = $\frac{33}{1}$ = 33 months.

This analysis shows that among the three networks, the mean recurrence of Globacom is higher followed by MTN and Airtel in that order.

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